Young Applied Analysts in the UK

University of Glasgow
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TITLES AND ABSTRACTS

Jonathan Bevan

Testing the stability of radial cavitating maps in nonlinear elasticity.

Some elastic materials form cavities in their interior when their boundary is stretched `enough'. Under such conditions, J. Ball showed that radial deformations corresponding to the formation of a single cavity are critical points of a polyconvex stored-energy function. This talk is about a condition, discovered in 2008 by J. Sivaloganathan and S. Spector, which is necessary for the `local minimality' of the radial cavitating maps. The condition itself is in the form of an inequality

$$\int_B \left|\operatorname{adj}\nabla w\left(\frac{w}{|w|^3}\right)\right|^q dx \geq \int_B \frac{1}{|x|^{2q}}\, dx$$

where $w : B \subset \mathbb{R}^3 \to \mathbb{R}^3$ are suitably defined maps, B is the unit ball in $\mathbb{R}^3$, and where $2q \in (2,3)$.

David Bourne

Numerical methods for optimal location problems

This talk treats a class of energies for particle systems where the particles interact nonlocally via a Wasserstein distance term. These types of energies, known as optimal location energies, arise in urban planning, electrical engineering, and materials science. Due to the nonlocal nature of these energies, minimising them numerically is difficult. I will show how this difficulty can be avoided by exploiting a deep connection between optimal transportation theory and computational geometry.

This is joint work with Mark Peletier, Steven Roper and Florian Theil.

Filippo Cagnetti

The rigidity problem for symmetrization inequalities

Steiner symmetrization is a very useful tool in the study of isoperimetric inequality. This is also due to the fact that the perimeter of a set is less or equal than the perimeter of its Steiner symmetral. In the same way, in the Gaussian setting, it is well known that Ehrhard symmetrization does not increase the Gaussian perimeter.

We will show characterization results for equality cases in both Steiner and Ehrhard perimeter inequalities. We will also characterize rigidity of equality cases. By rigidity, we mean the situation when all equality cases are trivially obtained by a translation of the Steiner symmetral (or, in the Gaussian setting, by a reflection of the Ehrhard symmetral). We will achieve this through the introduction of a suitable measure-theoretic notion of connectedness, and through a fine analysis of the barycenter function for a special class of sets.

These results are obtained in collaboration with Maria Colombo, Guido De Philippis, and Francesco Maggi.

Patrick Dondl

Energy estimates and relaxation for strain gradient plasticity

We consider a variational formulation of gradient elasto-plasticity subject to a class of hard single-slip conditions. Such side conditions typically render the associated boundary-value problems non-convex.

We first show that, for a large class of non-smooth plastic distortions, a given single-slip condition (specification of Burgers vectors) can be relaxed by introducing a microstructure through a two-stage process of mollification and lamination. The relaxed model can be thought of as an aid to simulating macroscopic plastic behavior without the need to resolve arbitrarily fine spatial scales.

We then apply this relaxed model to a specific system, in order to be able to compare the analytical results with experiments: A rectangular shear sample is clamped at each end, and is subjected to a prescribed horizontal or diagonal shear, modelled by an appropriate hard Dirichlet condition. We ask: how much energy is required to impose such a shear, and how does it depend on the aspect ratio? Assuming that just two slip systems are active, we show that there is a critical aspect ratio, above which the energy is strictly positive, and below which it is zero. Furthermore, in the respective regimes determined by the aspect ratio, we prove energy scaling bounds, expressed in terms of the amount of prescribed shear.

Federica Dragoni

Analysis on sub-Riemmanian geometries: why shall I care and can things go wrong?

The idea of the talk is to give a very basic introduction of what sub-Riemannian geometries are and what the Hörmader condition is.
I will try to give an overview of the ``magic powers'' of the Hörmander condition and explain why in several problems in analysis this can be the "best possible condition" to ask for.... but there are rainy days everywhere.

Yves van Gennip

PDEs on graphs: curvature and other curiosities

In image processing and data analysis, the data sets are often modelled as a graph in which the nodes represent the data points and the edges encode some relationship between the nodes, relevant to the task at hand. In recent years, people have studied classical continuum PDE models from image analysis, but formulated on graphs to be applicable to data analysis problems.

These studies show interesting connections between continuum results and the analogous problems on graphs. In this talk we will explore some of these PDE type problems formulated on graphs and their connections with both continuum results and notions from graph theory. Examples include the Allen-Cahn equation, threshold dynamics, and mean curvature on graphs, and their relation to graph cuts, mean curvature flow, and bootstrap percolation.

Aram Karakhanyan

Incompressible energy minimizers and the Monge-Ampère equation

We will consider a minimization problem with the hard Jacobian constraint in 2D.
Our main result will be a L^2 local estimate for the hydrostatic pressure p, provided that p>0. This will be done through rewriting the Euler-Lagrange equation as a Monge-Ampère type equation for a scalar potential function.

Matthias Kurzke

Vortex motion in ferromagnets

In thin ferromagnetic films, one can sometimes observe the emergence of vortex-like singularities. I will discuss how the standard evolution equation for ferromagnets, the Landau-Lifshitz-Gilbert equation, induces motion of these vortices. This is joint work with Christof Melcher (Aachen), Roger Moser (Bath), and Daniel Spirn (Minnesota).

Carlo Mercuri

Some quasilinear elliptic problems involving the critical Sobolev exponent

I will present some recent results on the variational analysis of some classes of boundary value problems involving the p-Laplace operator and nonlinearities of critical growth. We will focus on how to tackle the lack of compactness due to the critical Sobolev exponent and how to overcome some additional difficulties due to the singular/degenerate behaviour of the p-Laplace operator.

Tadahiro Oh

Invariant measures and probabilistic well-posedness for nonlinear dispersive PDEs

I will go over recent development on probabilistic approach to the study of nonlinear dispersive PDEs such as the nonlinear Schrodinger equations and describe how it allows us to go beyond the deterministic threshold.

Mariapia Palombaro

A discrete to continuum analysis of dislocations in nanowire heterostructures

Epitaxially grown heterogeneous nanowires present dislocations at the interface between the phases if their radius is big.
We consider a corresponding variational discrete model with quadratic pairwise atomic interaction energy. By employing the notion of Gamma-convergence and a geometric rigidity estimate, we perform a discrete to continuum limit and a dimension reduction to a one-dimensional system. Moreover, we compare a defect-free model and models with dislocations at the interface and show that the latter are energetically convenient if the thickness of the wire is sufficiently large.

Markus Schmuck

Upscaled phase-field equations for interfacial dynamics in strongly heterogeneous materials

We derive effective macroscopic Cahn-Hilliard equations [1,2] for general homogeneous free energies [3] which include the frequently applied double-well potential. The upscaling is done for perforated/strongly heterogeneous media. To the best of our knowledge, this is the first attempt of homogenizing nonlinear fourth-order equations in such domains. The new upscaled formulation should have a broad range of applicability due to the well-known versatility of phase-field equations. The additionally introduced feature of systematically and reliably accounting for confined geometries by homogenization allows for new modelling and numerical perspectives in both, science and engineering. One example is the effective description of wetting in porous media where one has a scale separation between the pore and the characteristic macroscopic length scale. Finally, we can characterize qualitatively the homogenized equations by error estimates.

This is joint work with Marc Pradas, Grigorios A. Pavliotis, and Serafim Kalliadasis (Imperial College, London).

[1] M. Schmuck, M. Pradas, G. A. Pavliotis, and S. Kalliadasis, Proc. R. Soc. A 468:3705-3724 (2012).
[2] M. Schmuck, M. Pradas, G. A. Pavliotis, and S. Kalliadasis, Nonlinearity 26:3259 (2013).
[3] M. Schmuck, G. A. Pavliotis, and S. Kalliadasis, Appl. Math. Lett. 35:12-17 (2014).

Dimitrios Tsagkarogiannis

Young-Gibbs measures for the Ising model

The value of the magnetisation at a macroscopic point of a magnetic material when observed at a mesoscopic scale can be realised either as a homogeneous state or as a fine mixture of the two pure phases of the system. This has been mathematically described with the use of Young measures. On the other hand, from an atomistic point of view at finite temperature, each such pure phase is described via an extremal Gibbs measure and mixtures via convex combinations of the extremal ones.

In this talk we connect the two descriptions deriving a macroscopic continuum mechanics theory for scalar order parameter starting from statistical mechanics. For this, we revisit a recent work by Kotecky and Luckhaus and we construct the so-called Young-Gibbs measures for the case of the nearest neighbour Ising model.

This is work in progress jointly with Alessandro Montino (GSSI) and Nahuel Soprano Loto (Buenos Aires).